Long-Run Timber Supply: Price Elasticity, Inventory Elasticity and the Use of Capital in Timber Production

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LONG-RUN TIMBER SUPPLY: PRICE ELASTICITY,
INVENTORY ELASTICITY, AND THE USE OF CAPITAL
IN TIMBER PRODUCTION

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ABSTRACT. Timber production requires substantially more capital per unit output than does virtually any other economic enterprise. The quantity of capital deployed depends primarily on the rotation length and the output price for timber. In a long-run timber supply model this gives rise to a "backward bending" supply curve. This paper summarizes a long-run model of timber supply, and computes the associated price and inventory elasticities. The role of capital in timber production is explored through a continuous-time formulation of the usual Faustmann point-input/point-output model. The theoretical results are illustrated through an example based on loblolly pine yields for the U.S. South.

KEY WORDS: Long-run timber supply, capital:output ratio.

This paper analyzes the long-run supply of timber, with particular attention paid to the role of capital. The model is long run in the sense that the time period of the analysis is adequate for the capital stock to adjust to the economically optimal steady-state level. The period of time necessary for this condition to be met depends on the initial age structure of the forest, the level of demand and the underlying biological productivity of the forest; it might range from less than a decade to more than a century.

This model of timber supply dates at least to Vaux's [1954] analysis of timber production in California, and has been used many times since: for the Douglas-fir region of the United States by the USDA Forest Service [1963] and Hyde [1980], for pine in the southern U.S. by Robinson [1980] and for the spruce-fir resource in Maine by Binkley

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The present analysis is both narrower and deeper than these other efforts. The rotation age is the only decision variable considered. To a great extent, the rotation age determines the quantity of capital used in timber production, so an analysis of capital logically focuses on this variable. In addition, the rotation length is perhaps the single most important variable determining the level of output from a forest (Davis [1976]). This analysis omits any formal discussion of management intensity or the amount of land devoted to timber production, although the concluding section comments on how these variables would influence the present results. Similarly, the nontimber products of the forest (recreation, water, environmental services) which may affect the optimal harvest period are excluded from consideration (see Hartman [1976] and Bowes, Krutilla, and Sherman [1984] or Bowes and Krutilla [1989] for discussions of how the presence of valuable nontimber forest products affects the analysis). Finally, this model focuses on the long-run steady-state supply of timber, and not the important problems of the transition to this steady state (Sedjo and Lyon [1990]; Berck [1976], Jugenfalt [1973]; Johansson and Löfgren [1985, Chapter 6]). My narrow perspective is adopted to focus on the role of capital in timber production, and is consistent with, for example, Samuelson's [1976] treatment of the problem.

The remainder of the paper is in four parts and a concluding comment. The first section details the long-run supply model. This section shows that the price elasticity of supply may be negative, indicating a backward bending supply curve for timber. Instability may arise in the long-run market equilibrium because of this unusual supply behavior.

Many short-run models of timber supply use the level of timber inventory to explain harvest levels. Section 2 below compares the inventory elasticity implied by the long-run model with the results from these studies.

The next section examines the capital-output ratio for the steady-state forest and clarifies the importance of capital in timber production.

The final section uses yield information for loblolly pine grown in the southeastern United States to demonstrate the practical significance of the theoretical results.
1. Long-run supply, price elasticity and market instability. The long-run timber supply model has two parts. The first describes the rotation decisions of forest owners as a function of timber price and other relevant parameters. This decision determines the level of timber production. The second explains precisely how the amount of timber produced depends on the rotation age, and therefore on price.

*Landowner behavior.* The forest owner selects the rotation age which maximizes the net present value \( \pi \) of timber receipts summed over an infinite planning horizon. Capital markets are perfect so the forest owner can lend and borrow at a constant, known interest rate \( i \) (equivalently, land markets perfectly reflect the present value of partially grown stands). Timber yield \( v \) per area is a known function of stand age \( t \); the yield function does not change over time. Regenerating a stand costs \( c \) per unit area, an amount which is constant through time. Lastly, the even-aged forest is regenerated promptly after clearcutting if it is profitable to do so.

These assumptions imply the rotation problem is stationary, so the forest owner solves

\[
\max_t \pi(t) = -c + pv(t)e^{-it} + \pi(t)e^{-it}.
\]

The stumpage price \( p \) is exogenous. The optimal rotation age \( t^*(p) \), and therefore the level of output, varies with price level.

The first order optimality conditions for \( t^*(p) \) can easily be found (see Jackson [1980]; Hyde [1980]; Chang [1983] or Johansson and Löfgren [1985, Chapter 4 and 5] by solving \( d\pi/dt = 0 \).

\[
\frac{\dot{v}}{v - \frac{c}{p}} = \frac{i}{1 - e^{-it}}
\]

where \( \dot{v} \equiv dv/dt \).

*Timber output.* Given the optimal rotation age, how much timber is produced? In the long run, capital can adjust to the economically desirable level. For timber production this implies that the forest has no timber older than \( r^*(p) \), and each year all timber reaching this age is harvested. Averaged over a rotation, the annual output of a forest of
area $A$ is $A v(t^*(p))/t^*(p)$. Without loss of generality, this paper takes $A = 1$, so the supply function is

$$s(p) = \frac{v(t^*(p))}{t^*(p)}.
$$

A "fully regulated" forest produces identically the long-run average annual output each year. This is achieved by an arrangement of age classes so that each occupies an area equal to $A/t^*$.

Before continuing, it will be helpful later in the analysis to note that supply is maximized at the rotation age which satisfies

$$\frac{\dot{v}}{v} = \frac{1}{t}.
$$

Equation (4) is simply a restatement of the forester's familiar maxim that average annual output is maximized when current annual increment ($\dot{v}$) equals mean annual increment ($v/t$). It is natural to call the rotation which satisfies (4) the maximum sustained yield (MSY) rotation.

**Long-run supply.** The supply model is illustrated in Figure 1. The top panel depicts the optimal rotation condition (2). The lower panel shows forest growth/steady state supply, equation (3). Given all the parameters of the model except price, the supply curve is defined by the following algorithm. For a given $p$, (2) is solved (upper panel of Figure 1) to find the optimal rotation age. Long-run supply then can be found from (3) (the lower panel of Figure 1). The process is iterated for various price levels until the supply curve is identified with suitable precision.

The supply function can also be developed in another way. First solve (2) for $p$ to link price directly with the optimal rotation age:

$$p = \frac{c}{v - \dot{v}(1 - e^{-it})/i}.
$$

Given a yield model $v(t)$, cost and interest rate, the supply curve can be constructed by examining a series of rotation ages. For each rotation age, (5) gives the price which makes that rotation optimal, and (3) gives the supply at that price.
FIGURE 1. Optimal rotation and timber supply.
The procedure is illustrated in Figure 2. The NW quadrant labelled $t^*(p)$ plots equation (5). The SW quadrant plots equation (3). The SE quadrant contains a 45° transfer line to map the average output from the SW quadrant onto the quantity axis of the supply curve, $s(p)$, which is shown in the NE quadrant.

Figure 2 shows the construction of the supply curve for three impor-
tant cases. In case (a), the price is so low that \( \pi = 0 \), and no long-run production takes place. Case (b) occurs at MSY. Binkley [1987] has shown that the MSY rotation occurs at a price of

\[
p = \frac{c}{v} \left[ \frac{1}{1 - (1 - e^{-at})/it} \right].
\]

Case (c) occurs at the quantity asymptote of the supply curve. To see this asymptote, refer to equation (2). Note that as \( p \to \infty \), the ratio of \( c/p \) approaches 0. The left hand side of equation (2) approaches \( \dot{v}/v \), and the supply curve grows increasingly inelastic at a price determined by the interest rate and the biological productivity of the forest.

From Figure 2, it is clear that the supply curve can have a negative slope. In general, what is the price elasticity of supply implied by this model? By definition, the price elasticity is

\[
\epsilon_p = \frac{ds}{dp} \cdot \frac{p}{s} = \frac{ds}{dt} \cdot \frac{dt}{dp} \cdot \frac{p}{s}.
\]

Since

\[
\frac{ds}{dt} = \frac{\dot{v}t - v}{t^2},
\]

then

\[
\epsilon_p = \frac{dt}{dp} \left[ \frac{\dot{v}}{v} - \frac{1}{t} \right] p.
\]

It is well known that \( dt/dp \) is negative, so the sign of the long-run supply elasticity depends on the sign of \( \dot{v}/v - 1/t \). If the price level is such that \( \dot{v}/v < 1/t \), then the price elasticity of supply is positive. Recall that this condition obtains only if the optimal rotation age is greater than the MSY rotation. Consequently, only in the unusual case where the economically optimal rotation is longer than MSY will the long-run supply curve have the usual positive slope. Otherwise, the supply curve will have a negative slope. The “backward bending” supply phenomenon has been noted by Clark [1976] for the case of fisheries, and by Hyde [1980] for the case of timber.

Before turning to questions of market equilibrium, note that if prices are so low that \( \pi < 0 \), no production at all will occur in the long run.
High enough costs or interest rates can clearly lead to a situation where all production occurs on the negatively sloped part of the supply curve. Thus, unlike Clark's [1976] fisheries example, the entire long-run timber supply curve might have a negative slope. This occurs because inputs are required to produce timber, where Clark [1976] takes the fishery to be wholly self-reproducing.

Market instability. For an open access fishery, Clark [1976] points out that the backward bending supply curve can lead to unstable market equilibria. Figure 3 shows the situation for competitive timber supply with three levels of demand. The lowest level of demand, \( D_1 \), corresponds to the usual sort of market equilibrium, and the stability
of $E_1$ depends on well-known elasticity and adjustment conditions. At a slightly higher level of demand, $D_2$, three market equilibria exist, and it is easy to see the $E_2''$ is unstable. Short-run timber demand is thought to be very inelastic. Long-run demand is likely to be more elastic but, if it remains fairly inelastic, then the price level associated with the equilibrium point $E_2''$ could be much higher than that associated with $E_2'$. The market instability implied by this analysis would then translate into dramatic price instability. Finally, if the market equilibrium settles at $E_2''$, the increase in demand from $D_1$ to $D_2$ is accompanied by a decrease in consumer's surplus.

2. Inventory elasticity. In this model timber supply is implicitly a function of timber inventory level. Many forest sector models use the level of timber inventory as a determinant of timber supply behavior (e.g., Adams and Haynes [1980], Binkley and Cardellicchio [1985], Newman [1987]). Consequently, it is of some interest to examine how supply responds to inventory level in this long-run model.

The inventory of a unit area, steady-state forest is

\[
I = \frac{1}{t} \int_0^t v(z) \, dz.
\]

This inventory can be increased in three general ways. The first two cases reflect exogenous changes in the inventory, where the third incorporates endogenous inventory adjustments.

The first way to alter the inventory adds to the forest another unit of land which is identical to that already in production. Alternatively, the yield function can be increased by a constant fraction at all ages. In both cases, it is obvious that the inventory elasticity is 1 because both supply (3) and inventory (10) are augmented by precisely the same amount.

The third, more interesting, case alters the inventory endogenously by changing some parameter—$i$, $c$, or $p$—so that the optimal rotation age changes. Then “apparent” inventory elasticity of supply can be calculated. This elasticity is termed an “apparent elasticity” because both the change in supply and the change in inventory are due to the exogenous change in some other parameter of the model.
By definition, the inventory elasticity $\varepsilon_I$ is

$$
\varepsilon_I = \frac{ds}{dI} \cdot \frac{I}{s}
$$

which can be rewritten

$$
\varepsilon_I = \left[ \frac{ds}{dt} \cdot \frac{1}{dI/dt} \right] \frac{I}{s}.
$$

From (10),

$$
\frac{dI}{dt} = \frac{v - I}{t}.
$$

Substituting the value of $ds/dt$ from (8) and rearranging gives

$$
\varepsilon_I = \left[ \frac{\dot{v}}{v} - \frac{1}{t} \right] \frac{\int_0^t v(z) \, dz}{(vt - \int_0^t v(z) \, dz)}.
$$

Because $\dot{v} > 0$ for all $t$, $vt > \int_0^t v(z) \, dz$ for all values of $t$, and the numerator of the second term in (14) is positive. Thus, the sign of the apparent inventory elasticity depends on the sign of the term in brackets. This term is positive if $t^* < MSY$, zero if $t^* = MSY$, and negative if $t^* > MSY$. The apparent inventory elasticity is positive for short rotations, falls to zero at $MSY$ and then becomes negative.

At any point along the long-run supply curve, the price and inventory elasticities will have opposite signs (except at $MSY$ where they are both identically zero). In contrast, most empirical supply studies take both elasticities to be positive.

Timber supply studies which employ inventory as an independent variable generally use one of two approaches to estimate the requisite elasticity. First, because time series data on timber inventory levels are frequently poor (and inventory would probably change only gradually over time in any case), it sometimes is not possible to obtain usable statistical estimates for an inventory term. In such cases the supply variable can be recast as the ratio of harvest to inventory, and the inventory variable omitted from the set of independent variables. This
specification implicitly constrains the inventory elasticity to be unity. To see this, consider a supply function specified as

\begin{equation}
\frac{s}{I} = f(p, X)
\end{equation}

or

\begin{equation}
s = f(p, X)I
\end{equation}

where \( X \) is a vector of nonprice independent variables though to affect supply. The inventory elasticity of supply in this model is

\begin{equation}
\frac{ds}{dI} \frac{I}{s} = f(p, X) \frac{I}{s} = 1.
\end{equation}

In some U.S. regions, Adams and Haynes [1980] use this specification for softwood timber supply. The Resource Information Systems, Inc. FORSIM softwood sector model uses this specification in all regions as does the model of the U.S. hardwood lumber sector developed by Binkley and Cardellichio [1985]. Using a unitary inventory elasticity is consistent with the first two kinds of inventory elasticities discussed above.

For some regions, Adams and Haynes [1980] were able to estimate softwood stumpage supply equations with inventory as an independent variable. For the regions where this was possible, they obtained estimates ranging from 0.2 to 1.46, with a preponderance of values near 0.5. As will be shown in Section 4, these results are not inconsistent with the third case examined if timber rotations are less than MSY.

3. Capital: output ratio. Because of the long time period involved in forest production, capital is a critical input. The capital stock required for a steady-state forest can be measured in several ways, and the present analysis uses perhaps the most conservative definition.

This definition can be developed most easily using a continuous time model of the timber production process. In each period the net income for a unit of forest can be decomposed into three parts:

\begin{align}
(18a) & \quad p\dot{p} = \text{gross income} \\
(18b) & \quad tpUV = \text{opportunity cost of the growing stock} \\
(18c) & \quad r = \text{land rent}.
\end{align}
In this context, the net present value function can be written as

\begin{equation}
\pi(t) = -c - \int_0^t p\hat{v}(z)e^{-iz} \, dz - \int_0^t i\nu(z)e^{-iz} \, dz - \int_0^t re^{-iz} \, dz.
\end{equation}

Before using these definitions to derive the capital output ratio, let us show that (19) is equivalent to (1). To do so, begin by integrating the first integral in (19) by parts:

\begin{equation}
\int_0^t p\hat{v}(e^{-iz}) \, dz = pv(t)e^{-it} + \int_0^t i\nu(z)e^{-iz} \, dz.
\end{equation}

Now substitute (20) into (19) to get

\begin{equation}
\pi(t) = -c + pv(t)e^{-it} + \int_0^t re^{-iz} \, dz.
\end{equation}

Efficient land markets imply that \( r \) adjusts so \( \pi = 0 \) (see Samuelson [1976] on this point). Integrating the last term of (21) and imposing this condition implies

\begin{equation}
\frac{r}{i} = \frac{-c + pv(t)e^{-it}}{1 - e^{-it}}.
\end{equation}

The term on the left hand side of 3.5, \( r/i \), is simply the capitalized value of land rents and corresponds to the economic rent we seek to maximize in (1). Thus, equation (19) is precisely the equivalent to the original problem described by equation (1). Casting the problem as a continuous input/continuous output problem gives precisely the same results as the more conventional point input/point output formulation.

The continuous formulation is useful because it highlights the role of capital in timber production. In this context, equation (18b) comprises the most limited definition of capital possible. This formulation treats regeneration costs as "labor," and land rental costs as "land."

In the steady-state forest the current annual value of the average capital deployed \( k \) is

\begin{equation}
k = \frac{ip}{t} \int_0^t v(z) \, dz.
\end{equation}
Capitalized over perpetuity at rate $i$, the capital deployed in a steady-state forest is

$$k = \frac{p}{t} \int_0^t v(z) \, dz.$$  

From equation (3), the annual income from the forest is

$$y = p \cdot s(p) = \frac{pv(t)}{t}.$$  

The capital:output ratio $k/y$ is then

$$\frac{k}{y} = \frac{\int_0^t v(z) \, dz}{v(t)}.$$  

Suppose that the discount rate $i$ increases. How does the capital/output ratio respond? The rotation will decrease (see, for example, Chang [1983]) and the value of the capital embodied in the inventory of growing stock will decline (see equation (13)). Output may increase or decrease with the change in rotation, so the direction of the change in $k/y$ is ambiguous. The loblolly pine example developed below shows that for most rotations of interest, $d(k/y)/dt$ is positive, so increases in interest rates will lead to reduction in the use of capital per unit output.

4. **An example: SI 80 loblolly pine.** While the foregoing theoretical analysis provides some definitive results concerning the nature of the long-run supply curve for timber, the practical importance of some of the theoretical concerns is not readily apparent. To provide a modest degree of empirical insight into the nature of this timber supply model, this section presents an example using yield information for Site Index 80 loblolly pine.

To ease the numerical burden of the example, the cubic foot/acre yields given by Schumacher and Coile [1960] were fit to a two-parameter yield equation

$$\ln[v(t)] = 8.8216 - 23.512/t \quad R^2 = 0.998$$

$$\begin{array}{cc}
(6.314) & (46.2) \\
n = 6.
\end{array}$$
The numbers in parentheses are the $t$-statistics for the null hypothesis that the associated coefficient is zero. For the purposes of this example, ordinary least squares regression produced an acceptable fit to the yield table.

For this example, regeneration costs $50/acre and the interest rate equals 0.025. Binkley [1985] has shown that for the long-run supply curve to have any positively sloping portion, the interest rate must be
less than the inverse of the $MSY$ rotation. Substituting the loblolly pine yield function (26) into equation (4) gives an $MSY$ rotation of 23.5 years, so the interest rate must be less than $1/23.5 = 0.043$ to show the general case of where the supply curve has first a positive slope at low prices, then a negative slope at higher prices. A comparatively low rate of $i = 0.025$ was taken in order to illustrate the general principles involved. Indeed, it is of some interest that more realistic interest rates would produce supply curves which are either almost perfectly inelastic or slope backwards throughout their entire range.

Figure 4 shows the long-run supply curve for this situation. This curve was constructed by solving equation (5) for $t = 1, 50$ in one year increments for the price which makes that rotation age optimal (for $t < 19$, the $p(t^*) < 0$). Supply at each price was determined from equation (3). Figure 4 plots the results of these calculations. The initial part of the curve slopes upward until the price rises to the point that $MSY$ is reached. Under assumptions of this example, $MSY$ occurs at a price of $0.082/\text{cf}$. The curve is asymptotic at a quantity of about 103.5 $\text{cf/acre/yr}$, or at a supply level which is about 98% of the $MSY$ supply. For $p < 0.0360, \pi < 0$ and no long-run production occurs. This price corresponds to an optimal rotation age of 30 years.

Figure 5 plots the price elasticity as a function of price level. This curve was derived numerically from equations (5) and (9). Nowhere is the supply curve very elastic. It is perfectly inelastic at $MSY$ and again at the quantity asymptote.

Figure 6 shows the "apparent" inventory elasticity as a function of the rotation age. At the quantity asymptote, the apparent inventory elasticity is 2.88, falling to 0.0 at $MSY$ and to -5.7 when timber production is no longer economic. The empirical results from the short-run timber supply studies cited in Section 2 above, fall close to the present results near $MSY$.

Finally, Figure 7 shows the capital:output ratio as a function of the rotation age. At $MSY$, the ratio is about 10 and at the quantity asymptote the ratio is about 7. In 1980, the ratio of reproducible fixed assets to value added for all U.S. manufacturing industry was 0.34 (UN, 1981, Tables U.S. 2.15 and U.S. 4.3). Even using perhaps the most restricted definition of capital possible, timber production requires two orders of magnitude more capital per unit output than does the U.S.
industrial sector as a whole.

Throughout the range shown in Figure 7, \(d(k/y)/dt\) is positive. As one would expect, increases in capital costs will lead to less capital used per unit of output. Until age seven, the converse is true, however.

5. Conclusion. Capital is a major component of forest production costs. The rotation length largely determines the quantity of capital used in a timber enterprise because the rotation age determines the amount of growing stock inventory. The rotation age also strongly influences the average output from the forest.

High stumpage prices imply not only that the output from the forest has a high value, but also that capital in the form of growing stock has a high opportunity cost. At high prices, it is optimal to conserve on the use of capital, and therefore to reduce the growing stock inventory by reducing the rotation age.
FIGURE 6. Inventory elasticity, loblolly pine ($c = \$50/a, i = 0.025$).

This kind of capital conservation can also reduce forest growth. In the long run, timber supply can therefore fall as a consequence of higher timber prices. Higher output prices inevitably mean higher capital costs, and the timber supply curve bends backward as a result of the necessity to reduce these costs. Market instability may result from this unusual cost structure for timber production.

These conclusions are, of course, strictly correct only under the many assumptions necessary for such concise and unambiguous results. Two important market adjustments have been ignored in the analysis: changes in management intensity and changes in the area of land devoted to timber production. Both kinds of adjustments make timber supply more elastic than found here, particularly at low prices where there is much latitude for intensifying silvicultural practices or for extending the intensive and extensive margins of timber production. As a forest sector develops, however, the opportunity for these adjustments
FIGURE 7. Capital output ratio, SI 80 loblolly pine ($c = \$50/a$, $i = 0.025$).

will diminish, and capital will tend to dominate the timber production cost structure. The structural problems for the sector associated with inelastic or backward bending supply will then appear.

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